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ASSESSING THE AFFORDANCES OF SIMREAL+ AND THEIR APPLICABILITY TO SUPPORT THE LEARNING OF MATHEMATICS IN TEACHER EDUCATION

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ABSTRACT

Aim/Purpose	Assess the affordances and constraints of SimReal+ in teacher education
Background	There is a huge interest in visualizations in mathematics education, but there is little empirical support for their use in educational settings
Methodology	Single case study with 22 participants from one class in teacher education. Quantitative and qualitative methods to collect students' responses to a survey questionnaire and open-ended questions
Contribution	The paper contributes to the understanding of affordances and constraints of visualization tools in mathematics education
Findings	The visualization tool SimReal+ has potential for learning mathematics in teacher education, but the user interface should be improved to make it more usable for different users. Teachers need to consider technological and pedagogical affordances of SimReal+ at the student, classroom, and mathematics subject level
Recommendations for Practitioners	Address technological and pedagogical affordances of SimReal+
Recommendation for Researchers	Improve the design of SimReal+ to make it technologically and pedagogically more usable
Impact on Society	Understand the affordances and constraints of visualization tools in education
Future Research	Implement a next cycle of experimentation with SimReal+ in teacher education to ensure more validity and reliability

Keywords	pedagogical affordances, technological affordances, SimReal+, visualization tool

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INTRODUCTION

There is a huge interest in visualizations in mathematics education, but there are few empirical studies in authentic educational settings. SimReal+ is a new visualization tool that is used to teach a wide range of mathematical topics at the undergraduate level, but the suitability of the tool for supporting mathematical learning in teacher education has not been fully evaluated. The adaptability of digital tools is a complex issue, and there is a need for acceptance models like the one developed for education and other domains (Zogheib, Rabaa'i, Zogheib, & Elsaheli, 2015). A wide range of theoretical approaches can be used to address and evaluate the use of digital tools in mathematics education (Bokhove, & Drijvers, 2010; Drijvers, Kieran, & Mariotti, 2010). No one of these approaches is ready-made to assess the affordances of SimReal+ and their applicability to support the learning of mathematics and explore the extent to which the tool is useful in teacher education (Geiger, Forgasz, Tan, & Calder, 2012). The theory of affordances provides a powerful framework to address the suitability of SimReal+ in teacher education at the ergonomic, functional, student, classroom, and subject level. This fits well with the way mathematics is taught in teacher education using digital tools. The article is structured as follows. Firstly, the visualization tool SimReal+ is presented. Secondly, the theoretical framework is outlined. This is followed by the research questions and methods used to collect and analyze data. The results are then presented and discussed. Finally, some remarks on future research conclude the article.

VISUALIZATIONS IN MATHEMATICS EDUCATION

THE NOTION OF VISUALIZATION

Basically, visualization is the ability to use and reflect upon pictures, graphs, animations, images, and diagrams on paper or with digital tools with the purpose of communicating information, thinking about and advancing understandings (Arcavi, 2003). There is a huge interest in visualizations in mathematics education (McKenzie, & Clements, 2014; Presmeg, 2014). Textbooks are filled with pictures, diagrams, and graphs. Graphing calculators have become integral part of mathematics education. But, there is little empirical support for using visualization tools in mathematics education (Macnab, Phillips, & Norris, 2012). There is also no clear definition of the notion of visualization. Presmeg (2014) includes processes of construction and transformation of both visual mental images and inscriptions of a spatial nature that may be implicated in doing mathematics. According to Arcavi (2003), visualizations incorporate both the process and product of creation, and reflection upon pictures, images in the mind (internally), and on paper or with digital tools (externally). Another use of the notion is visual representation. Visualizations are also used in relation to the terms "representation" and "inscriptions", and some researchers do not make a difference between mathematical visualizations (pictures, images, and diagrams) and mathematical representations (verbal, graphical, and symbolical).

THE VISUALIZATION TOOL SIMREAL+

SimReal+ is an interactive visualization tool for teaching and learning mathematics for a number of mathematical subjects. SimReal+ uses a graphic calculator, video lessons, video live streaming, video simulations, and interactive simulations to teach mathematics (P. H. Hogstad, n.d.). SimReal+ has more than 5000 applications and tasks in various areas of mathematics (Brekke, & P.H. Hogstad, 2010). The tool can be divided in small subsets, while keeping the same structure and basic user interface. According to N. M. Hogstad, Ghislain, & Vos (2016), a subset of SimReal+ called Sim2Bil provides 4 windows for visualizations: simulation, graph, formula, and menu window.

There are few studies on SimReal+ in the research literature. Three studies focused on teaching mathematics in higher education (Brekke, & P.H. Hogstad, 2010; Gautestad, 2015; N. M. Hogstad, 2012). These studies report on positive attitudes towards the use of SimReal+ and its usefulness in difficult and abstract mathematical areas. Students considered SimReal+ as a positive supplement to ordinary teaching and encountered few challenges. Another study addressed the use of SimReal+ in an upper secondary school (Curri, 2012). It reported on positive students' attitudes towards the use of the tool in classroom. Some students did not found visualizations very useful and that integrating the tool into the curriculum was not simple. These studies used quantitative and qualitative methods to assess students' perceptions of SimReal+. Finally, N. M. Hogstad et al. (2016) studied a subset of SimReal+ called Sim2bil that aims at exploring how engineering students use visualizations in their mathematical communication.

THEORETICAL FRAMEWORK

A wide range of theoretical approaches can be used to address the usefulness of digital tools in mathematics education (Artigue, Cerulli, Haspekian, & Maracci, 2009; Drijvers et al., 2010). No one of these approaches is ready-made to address the purpose of this work. SimReal+ is considered as a positive supplement to ordinary teaching at the undergraduate level, but the use of the tool in teacher education has not been fully investigated to determine its ultimate value. The concept of affordances is used as a theoretical framework to explore the extent to which the tool affords students' engagement with mathematics.

THE CONCEPT OF AFFORDANCES IN THE RESEARCH LITERATURE

The term "affordance", originally proposed by the perceptual psychologist James J. Gibson in his book "*The Ecological Approach to Visual Perception*" (1977) refers to the relationship between an object's physical properties and the characteristics of a user that enables particular interactions between user and object. An affordance is a relation between an organism and object with the object perceived in relation to the needs of the organism: cliffs afford falling off, small stones afford throwing, chairs afford sitting, door afford opening, etc. According to Gibson (1977), affordances emerge in perception from the relationship between observer, object, and environment. They refer to action possibilities, that is, what the perceiver can do with the object.

The concept of affordances was introduced to the Human-Computer-Interaction community by Donald Norman in his book "*The Psychology of Everyday Things*" (1988). Accordingly, the term "affordance" refers to the perceived and actual properties of the thing, primarily those fundamental properties that determine just how the thing could possibly be used. According to Norman (1988, pp. 9), affordances provide strong clues to the operations of things. When affordances are taken advantage of, the user knows what to do just by looking: no picture, label, or instruction needed (Norman, 1988). In contrast to Gibson, Norman was interested in how the environment could be designed, in perception of the user (rather than the invariants actual properties of the thing), and how goals, culture, and past experiences influence perception.

Based on Norman's ideas, Turner and Turner (2002) defined a three-layer articulation of affordances. Perceived affordances refer to how a task is tackled in terms of effectiveness, efficiency, and satisfaction. Ergonomic affordances refer to the actions involved in the tasks, and cultural affordances refer to the cultural teaching/learning objectives.

Kirchner, Strijbos, Kreijns, and Beers (2004) specified a three-layer definition of affordance. Firstly, technological affordances are properties of digital tools that are linked to usability. Secondly, educational affordances are properties of tools that act as facilitators of teaching and learning, and, finally, social affordances are properties of tools that act as social facilitators. Similarly, Conole (2013) used the concept of affordances and discussed it in relation to different information technologies and the complex and dynamic co-evolving relationship between technologies and users.

In mathematics education, using an Activity Theory perspective, Chiappini (2012) applied the notions of perceived, ergonomic, and cultural affordances to Alnuset, a digital tool for high school algebra. Finally, Watson (2004, 2007) applied the notion of affordances to the design of mathematical tasks.

Summarizing, digital tools mediate between the individual, technological, pedagogical, and socio-cultural contexts. They provide learning opportunities at the technological, pedagogical, and socio-cultural level. As a result, affordances of digital tools are situated within the interrelationships between the learner, the technological, pedagogical, and socio-cultural level.

Affordances of SimReal+

Based on the research literature and the specificities of mathematics education, this work proposes two types of affordances at five different levels for SimReal+ (Figure 1):

- Technological affordances at the ergonomic level
- Technological affordances at the functional level
- Pedagogical affordances at the student level or mathematical task level
- Pedagogical affordances at the classroom level or student-teacher interaction level, and
- Pedagogical affordances at the subject level, that is the area of mathematics being taught

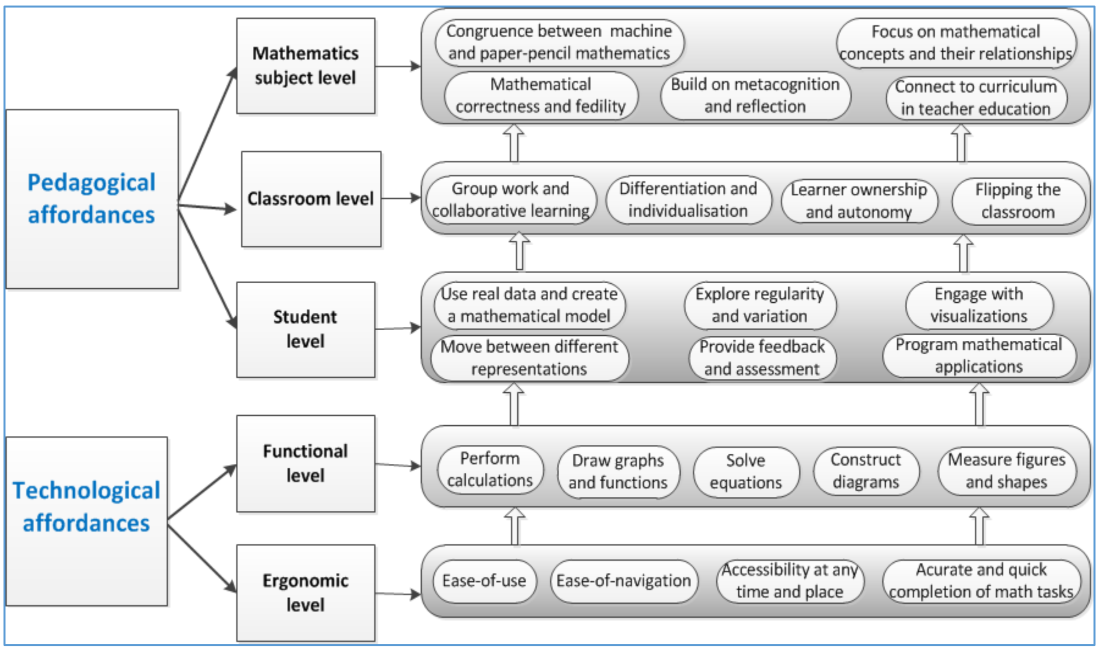

Figure 1. Affordances of SimReal+

Technological affordances can be provided at the ergonomic and functional level. At the ergonomic level, these are ease-of-use, ease-of-navigation, accessibility at any time and place, and accuracy and quick completion of mathematical activities. From the functional point of view, SimReal+ helps to perform calculations, draw graphs and functions, solve equations, construct diagrams, and measure figures and shapes. Pedagogical affordances are supported by technological affordances.

There are a number of pedagogical affordances that can be provided at the student level, e.g., using the tool to freely build and transform mathematical expressions that support conceptual understanding of mathematics, such as collecting real data and create a mathematical model, using a slider to vary a parameter or drag a corner of a triangle in geometry software, moving between symbolic, numerical, and graphical representations, simulating mathematical concepts, or exploring regularity and change (Pierce, & Stacey, 2010). At this level, the motivational factor is important, especially when using visualizations to engage students in mathematical problem solving. Furthermore, feedback in various forms to students' actions may foster mathematical thinking. Programming may also be a way of using SimReal+ to acquire mathematical understanding.

Likewise, a number of pedagogical affordances are provided at the classroom level (Pierce, & Stacey, 2010). This results in changes of interpersonal dimensions, such as change of teachers' and students' role, less teacher-directed and more student-oriented instruction, and using SimReal+ as "new" authority in assessing learning, resulting in students taking greater control over their own learning (learner autonomy). Other affordances at this level are change of social dynamics and more focus on collaborative learning and group work, as well as change of the didactical contract (Brousseau, 1997). Variation in teaching and differentiation are other opportunities offered by digital tools at this level (Hadjerrouit, & Bronner, 2014). This may result in flipping the classroom, which is another way of using SimReal+ at this level.

Finally, three types of pedagogical affordances can be provided at the mathematical subject level (Pierce, & Stacey, 2010). The first one is fostering mathematical fidelity, looking at congruence between machine mathematics and ideal or paper-pencil mathematics, and promoting faithfulness of machine mathematics (Zbiek, Heid, Blume, & Dick, 2007). The second affordance is amplifying and reorganizing the mathematical subject. The former is accepting the goals to achieve those goals better. Reorganizing the mathematical subject means changing the goals by replacing some things, adding others, and reordering others. For example, in calculus there might be less focus on skills and more on mathematical concepts (Pierce, & Stacey, 2010). In geometry, there might be emphasis on more abstract geometry, and away from facts, more argumentation and conjecturing (Pierce, & Stacey, 2010). Likewise, it may be useful to support tasks which encourage metacognition, e.g., starting with real-world applications, and using SimReal+ to generate results. Finally, connecting SimReal+ to the curriculum in teacher education is also an opportunity offered at this level to teach school mathematics.

RESEARCH STRATEGY

THE SETTING OF THE STUDY

The study was conducted at the University of Agder in the context of a technology-based course for teacher educators. In addition to SimReal+, the course covered various digital tools and their didactical use in classroom such as Excel, GeoGebra, Aplusix, and diverse pedagogical software in mathematics education. Twenty-two teacher students *(N=22)* were enrolled in the course in 2015. The students had very different knowledge background in mathematics ranging from primary to secondary mathematics. They had also varied experience in using digital tools. The minimum requirement was the completion of a bachelor degree in teacher or mathematics education. None of the students had any prior experience with SimReal+.

RESEARCH QUESTIONS

This work aims at investigating the students' perceptions and engagement with the visualization tool SimReal+ in terms of affordances, limitations, and gains of the tool in teacher education. It addresses three main research questions:

a) How do students perceive SimReal+ in terms of affordances?
b) How does SimReal+ afford students' engagement with mathematics?
c) What are the limitations and gains of SimReal+ in teacher education?

TEACHING ACTIVITIES

Teaching activities over a period of three weeks were designed and analyzed according to the theoretical framework and research questions. The activities included video lectures, visualizations, and simulations of basic, elementary, and advanced mathematics using SimReal+ and diverse online teaching material. Table 1 gives an overview of the teaching activities.

Table 1. Teaching activities

TEACHING ACTIVITY	TEACHING AIDS	SUBJECT	DATE	DURATION
Exercises in basic and elementary mathematics	• Online teaching material • SimReal+	• Introduction to SimReal+ • Basic mathematics: games, dices, tower of Hanoi, and prison • Elementary mathematics: Multiplication, algebra, Pythagoras, and reflection	Week 34	3 hours
Exercises in basic and elementary mathematics	• SimReal+	• Basic mathematics: games, dices, tower of Hanoi, and prison • Elementary mathematics: Multiplication, algebra, Pythagoras, and reflection	Week 34	2 hours
Exercises in advanced mathematics	• SimReal+ • Online teaching material	• Advanced mathematics: measurement, trigonometry, conic section, parameter, differentiation, and Fourier	Week 35	3 hours
Exercises in advanced mathematics	• SimReal+ • Online teaching material	• Advanced mathematics: measurement, trigonometry, conic section, parameter, differentiation, and Fourier	Week 35	2 hours
Evaluation of SimReal+	• Survey questionnaire	• Evaluation of SimReal+ using a survey questionnaire and open-ended questions	Week 36	1-2 hours

METHODS

This work is a single case study in teacher education. The participants (*N=22*) were teacher students from one class. The study aims at exploring the use of SimReal+ for mathematical learning in a technology-based course in teacher education. The research questions are exploratory in nature. By exploring the use of SimReal+ from the affordance perspective, some opportunities and constraints for the learning of mathematics can be revealed. Both quantitative and qualitative methods were used to collect and analyze students' responses to a survey questionnaire with open-ended questions. The design of the survey was guided by the theoretical framework and research questions.

To measure the students' perceptions of SimReal+, a survey questionnaire with a five-point Likert scale from 1 to 5 was used, where 1 was coded as the highest and 5 as the lowest (1 = "Strongly Agree"; 2 = "Agree"; 3 = "Neither Agree nor Disagree"; 4 = "Disagree"; 5= "Strongly Disagree"). The average score (MEAN) was calculated, and the responses to open-ended questions were analyzed qualitatively. The survey included 72 statements that were distributed as follows: technological affordances (12), pedagogical affordances at the student level (22), classroom level (18), and mathematical subject level (20). The students were asked to respond to the survey using the five-point Likert scale and to comment on each of the statements in their own words. In addition, the students were asked to provide written answers in their own words to open-ended questions. The presentation of the comments and responses to open-ended questions were guided by technological and pedagogical affordances of the theoretical framework, and some open-coding to bring to the fore information that was not covered by the framework, but related to digital tools in mathematics education.

RESULTS

HOW DO STUDENTS' PERCEIVE SIMREAL+ IN TERMS OF AFFORDANCES?

Guided by the affordances of SimReal+, a subset of the most important students' responses is given in this section. These are quantitative results based on the five-point Likert scale from 1 to 5. Table 2 shows the results achieved relative to technological affordances.

Table 2. Technological affordances

STATEMENT	MIN	MAX	MEAN	SD
SimReal+ is user-friendly and allows a quick familiarization with the tool	2	5	3.23	0.869
SimReal+ is easy to start and exit	1	4	2.36	1.002
SimReal+ is accessible anytime and anywhere	1	5	2.27	1.032
SimReal+ has readily available mathematical content	1	4	2.05	0.844
It is not easy to navigate through SimReal+ pages	1	5	2.86	1.082
SimReal+ video lessons and live streaming are of good quality	1	4	2.27	0.827

The results show that the vast majority of the students indicated that SimReal+ lacks a user-friendly interface, even though it is easy to start and exit. For many students, the tool was accessible anywhere and anyplace, but the navigation through the tool was not straightforward. On the positive side, SimReal+ has a ready-made mathematical content and the video lessons and live streaming are of good quality. This is reflected in many students' responses. However, there is a large standard deviation (SD) highlighting a large amount of variation among the participants.

Table 3. Pedagogical affordances at the student level

STATEMENT	MIN	MAX	MEAN	SD
SimReal+ visualizations and animations can be used as a motivational factor to learn mathematics	1	5	2.50	1.144
SimReal+ mathematical visualizations are easy to understand	1	4	2.18	0.664
SimReal+ visualizations and simulations provide opportunity to help me gain knowledge that is otherwise difficult to acquire	2	4	2.64	0.790
SimReal+ combination of video lessons, visualizations, live streaming of lessons, exercises, practical applications help me to understand and connect mathematical concepts	1	3	1.77	0.685
SimReal+ is helpful to refresh mathematical skills and knowledge by watching video lessons and visualizations	1	4	2.09	0.811
SimReal+ presents the mathematical content in several ways (text, graphs, visualizations, simulations, etc.)	1	4	1.68	0.780
SimReal+ facilitates various activities with the students (problem solving, video lectures, live streaming, exercises, etc.)	1	3	1.82	0.588

In terms of pedagogical affordances at the student level (Table 3), many students think that SimReal+ provides motivation for doing mathematics, and it engages them in mathematical problem solving, particularly when using mathematical visualizations. These are useful to gain mathematical knowledge that is otherwise difficult to acquire. Most students liked very much the combination of live streaming of lessons, video lectures, simulations, and animations. Furthermore, SimReal+ is helpful to refresh students' mathematical knowledge. Likewise, SimReal+ also provided variation in the way mathematics is taught. In terms of assessment, most students agreed that SimReal+ does not provide sufficient feedback, responses, hints, or review modes to their actions. Table 3 shows the results for pedagogical affordances at the student level.

In terms of pedagogical affordances at the classroom level (Table 4), the vast majority of the students think that SimReal+ does not support much cooperation or group work, and it does not have collaborative tools integrated into it. However, many students agreed that they can use SimReal+ on their own, and it is not controlled by the teacher, and they do not need much help from the teacher or textbooks to solve exercises. In contrast, most students think that the tool can be used as alternative to textbooks, or supplement to textbooks and lectures. In terms of differentiation and individualization, many students believed that the level of difficulty of mathematical tasks is high, and it is relatively difficult to adjust the tool to the students' knowledge level. On the positive side, many students think that they can work at their own pace when using SimReal+. Note that there is a large standard deviation (SD) in most responses, which means that the answers are away from the mean. This reflects a large amount of variation in the student group being studied.

Table 4. Pedagogical affordances at the classroom level

STATEMENT	MIN	MAX	MEAN	SD
SimReal+ stimulates students to cooperate	2	4	2.91	0.684
Collaboration tools are integrated into SimReal+	2	5	3.38	0.921
SimReal+ contains collaborative and group tasks	1	5	3.36	1.049
I can use SimReal+ on my own	1	5	2.32	1.171
The work with SimReal+ is mostly controlled by the teacher	1	5	3.10	0.995
I do not need to ask the teacher for help when I use SimReal+	1	5	3.18	1.053
I need to use mathematics textbooks when I use SimReal+	2	5	3.23	0.869
SimReal+ visualizations and animations can be used as an alternative to achieve variation in teaching mathematics	1	2	1.45	0.510
SimReal+ can be used to supplement mathematics textbooks	1	5	2.00	0.926
SimReal+ can be used to supplement lectures in classroom	1	3	1.77	0.528
SimReal+ contains multiple levels of difficulty, and can be adapted to different students' knowledge levels	1	5	2.91	1.019
SimReal+ provides opportunities for the teacher to make individual adjustments for each student, and define exercises	1	4	2.82	0.853
SimReal+ enables students to work at their own pace	1	4	1.91	0.750

In terms of pedagogical affordances at the mathematical subject level (Table 5), most students agreed that SimReal+ provides a high quality of mathematical content. Many students also found that SimReal+ is mathematically correct, and that the tool can display mathematical formulas, functions, graphs, numbers, and geometrical figures, in some contrast to the score achieved in terms of congruence with paper-pencil techniques. Furthermore, many students believed that SimReal+ is appropriate to use in secondary schools and, in a lesser degree, in middle and primary schools. Nevertheless, the vast majority of the students think that the tool enables the teacher to concretize the mathematical subject curriculum. Note that there is a large standard deviation in most responses. Table 5 shows the results.

Table 5. Pedagogical affordances at the subject level

STATEMENT	MIN	MAX	MEAN	SD
SimReal+ has a high quality of mathematical content	1	4	1.73	0.703
The underlying properties, representations, operations, and procedures of SimReal+ are mathematically correct	1	4	2.14	0.774
SimReal+ displays formulas, functions, graphs, numbers, and geometrical figures in a correct way	1	3	2.05	0.575
SimReal+ enables me to apply my own paper-and-pencil reasoning steps when solving mathematical problems	1	5	2.73	0.985
SimReal+ is an appropriate tool to use in secondary schools	2	5	2.36	0.790
SimReal+ is an appropriate tool to use in middle schools	2	5	2.86	0.889
SimReal+ is an appropriate tool to use in primary schools	1	5	3.05	1.327
SimReal+ enables the teacher to concretize the mathematics subject curriculum	1	3	2.05	0.653

Summarizing, the quantitative results show that SimReal+ still lacks a user-friendly interface. It motivates many participants to do mathematics at the student level, and it provides variation. The results are less promising in terms of pedagogical affordances at the classroom level, except using SimReal+ as supplement to lectures and textbooks, alternative to achieve variation, and working at their own pace. At the subject level, the tool has a high quality of mathematical content. The mathematical notations, representations and formulas are correct, and many students believed that the tool is appropriate at the secondary school level.

How does SimReal+ afford Students' Engagement with Mathematics?

This is both a quantitative and qualitative presentation of the comments and responses to open-ended questions. It is guided by the theoretical framework in terms of affordances and some open-coding to discover information that was not covered by the theoretical framework, but related to issues of digital tools in mathematics education.

The full set of issues and criteria are listed in Table 6, 7, 8, 9, and 10 together with the number of comments made by distinct students making such comments. The comments made by the students are described for the five levels of the framework, and supplementary issues that were addressed by open-ended questions.

Technological affordances

The most important technological affordances in terms of number of comments addressed by the students were ease-of-use, template, programming, user manual, and navigation. Table 6 gives an overview of the number of comments made by distinct students. As the table shows, navigation is the most frequent item addressed by the students, followed by ease-of- use and structure of menu, which are crucial in terms of technological affordances.

The written comments indicate that students were globally satisfied with the technological affordances in terms of easy-to-start, accessibility, management facilities, readiness of mathematical content, video life streaming, and, in lesser degree, with navigation. However, the tool is not considered as easy-to-use, since most students don't think that SimReal+ has an intuitive user interface that helps to navigate easily through the tool. SimReal+ has many buttons and choices so that it could be difficult to know which option is preferable. This is evident in one student's comment: *"it is rather hard to figure out how you can fiddle with them, for example, how to manipulate parameters (…). Moreover, in some dynamic simu-*

lations there is only a limited space where you can see the movement that takes place". Other students' comments support the view that SimReal+ is not sufficiently useful without a user manual. This is in line with the results achieved by the survey questionnaire.

Table 6. Technological affordances

ITEM	NUMBER OF COMMENTS	NUMBER OF DISTINCT STUDENTS
• Ease-of-use	20	20
• Accessibility	3	3
• Structure of menu	17	17
• User manual	16	16
• Navigation	26	22

Pedagogical affordances at the student level

In terms of pedagogical affordances at the student level, the most important issues addressed by the students were as follows: support for doing mathematics, motivation, variation, feedback, programming, and templates. Table 7 shows the number of comments addressed by distinct students. As the table shows, the level of engagement is the most frequent item, followed by use of templates, multiple representation of mathematics, and moving between different representations. Except the use of templates, which is important for programming mathematical functions, the other items are important from a pedagogical point of view.

Table 7. Pedagogical affordances at the student level

ITEM	NUMBER OF COMMENTS	NUMBER OF DISTINCT STUDENTS
SUPPORT FOR DOING MATHEMATICS		
• Moving between different representations	12	12
• Simulating mathematical concepts	6	5
MOTIVATION		
• Demonstrations	6	5
• Level of patience	14	14
• Engagement with visualizations	33	22
VARIATION		
• Varied teaching	6	6
• Multiple representations of mathematics	17	11
• Support teaching	6	6
• Learning outcomes	2	2
• Understanding	3	3
FEEDBACK		
• Hints	6	6
• Formative assessment	7	7
• Summative assessment	10	10
• Tutorial	3	3
PROGRAMMING AND USE OF TEMPLATES		
• Programming mathematical visualizations	3	3
• Use of templates	19	10

Most students believed that SimReal+ provides support for doing mathematics in terms of moving between different representations and simulating mathematical concepts. This is evident in the vast majority of the comments. One student pointed out that SimReal+ is "*absolutely essential*" when it connects video lessons, simulations, live streaming of lessons, and exercises, because it "*can help students to understand and see the connections between different mathematical themes*", as another student indicated. Likewise, most students revealed that SimReal+ visualizations provide opportunity to help them gain knowledge that is otherwise difficult to acquire with traditional means.

Furthermore, many students think that the menu and structure of the user interface are not well designed to support engagement with visualizations as this representative comment clearly reveals: *"is not optically nicely designed, but how this effects the learning depends on the age/level of studies (…). E.g. that would rather be a problem for primary school children, but not for university students, who are more conscious and directed to the content of the course they learn".* Nevertheless, the visualisations in themselves are engaging for the vast majority of the students and that the "*simulations are highly stimulating and pose a problem in a much exciting way that verbal description would do".*

Specific issues concern students' engagement and feeling of wonder when working with visualizations, usefulness of demonstrations and explanations of mathematical concepts, level of patience when working with animations, and difficulty of navigation. Most students liked animations that trigger wonder, because the level of engagement and curiosity is increased. These in turn support the learning of mathematics. Likewise, demonstrations of mathematical concepts are needed because students learn more by using visualisations and animations. In contrast, the level of patience is low for most students, especially when the animation does not work properly, even though patience is context dependent. Finally, navigation must be improved to make it more intuitive and simple. "*Less is more*!" was one of the students' comments.

In terms of variation and multiple representations of mathematical content, most students agreed that SimReal+ is fully appropriate to use as an alternative to achieve variation in teaching mathematics as this comment clearly shows: "*It is a great tool that can contribute to teaching and amplify the learning outcome and the comprehension of notions (for example in the case of graphs of functions and derivatives (…)".* Another student thinks that *"this is the best part of SimReal+. It offers a lot of opportunities to work with visualisations and animations".* This in turn may support the understanding of mathematics and help to improve learning outcomes.

In terms of feedback, SimReal+ does neither provide a diagnosis of student's problem solving nor appropriate feedback that is adapted to the students' knowledge level. In addition, SimReal+ does neither build student profiles nor serve up appropriate questions or several questions to the students. Furthermore, SimReal+ does neither have a review mode showing what the student has done wrong or right, nor allow for the use of several question types. The lack of student profiles may prevent participants from engaging in authentic learning activities with SimReal+. Hence, it is obvious that the role of the teacher is still important in assessing the students' learning of mathematics. The following comment summarizes the limitations of SimReal+ in terms of formative assessment: "*Feedback in each attempt must be (…) ameliorated. Where one fails to provide a correct answer, SimReal+ could "gauge" the nature of it and provide the proper feedback/help".*

The students were asked in what way do they think programming mathematical visualizations by their own will help them in understanding mathematics if they can use different templates without any special tool. They just need to write the code directly into a Web page so that the concentration can be on mathematics and not on difficult details in the programming process.

Most students agreed that templates can help to concentrate on the mathematical part, and "*investigate empirically properties of math structures that lie behind the visual representations*", "*help to explore mathematics realities in ways pen and paper can't*", and "*force to think through how the concepts really are*". Likewise, most students think that it is a great idea to design such templates so that they can be able by their own to program mathematics (both elementary and advanced visualizations and simulations) directly into their own Web pages without any special tool. Most students liked the idea of using templates, as this comment clearly highlights: "*Would be a great idea to have easy access in such a thing*".

Pedagogical affordances at the classroom level

In terms of pedagogical affordances at the classroom level, the most important issues addressed by the students in terms of number of comments are collaboration, individualisation and differentiation, student autonomy, and flipped classroom. Table 8 shows the number of comments addressed by distinct students. The most frequent item is student knowledge level, followed by group work and flipped classroom, and level of difficulty of mathematical tasks. It appears that individualisation/differentiation is the most important criterion in terms of number of comments.

Table 8. Pedagogical affordances at the classroom level

ITEM	NUMBER OF COMMMENTS	NUMBER OF DISTINCT STUDENTS
COLLABORATION		
• Group work	14	14
• Collaboration tools	5	5
INDIVIDUALISATION/DIFFERENTIATION		
• Student knowledge level	15	15
• Work at own pace	6	6
• Level of difficulty of mathematical tasks	12	12
STUDENT AUTONOMY		
• Ownership	5	5
• Need for teacher help	8	7
• Supplement to textbooks	2	2
• User manual	6	6
FLIPPED CLASSROOM	14	14

In terms of collaboration, the vast majority of the students think that the tool supports individual problem solving rather than group work. Moreover, the tool does not have deliberate collaborative or communication tools installed: Nevertheless, one student noted that *"some simulations stimulate communication in a sense that one might want to take ideas from another, e.g., (…), how somebody else perceives the visual illustrations and work with them"*.

Then, the students indicated that SimReal+ does not provide a sufficient level of differentiation, individual adjustment, and choice of level of difficulty. Nevertheless, some students agreed that the tool enables them to work at their own pace, which is a motivational factor in keeping them engaged in mathematical problem solving. In contrast, the tool does not easily allow students to customize the tool according to their knowledge level, as this rather critical comment reveals: "*There exit different types of exercise that have diverse level of difficulty (….). But, when it comes to each category specifically there is not really a variety of scaled-difficulty exercises. For example, there are no equations that would need more technical, sophisticated algebraic manipulations, but only very simple ones"*.

Furthermore, some students believed that the tool does not provide ownership and more autonomy to do mathematics, as this rather critical comment shows: "*SimReal+ provides little mathematical background description of the simulations. Even though formulas are provided in order to reach a generality, one would need teachers or textbooks"*. However*," the possibility to meddle with the simulations offers the opportunity to automatically develop the understanding of notions",* as another student indicated. Finally*,* eight students also still think that SimReal+ does not fully allow students to work independently from the teacher. Textbooks and user manuals are still needed when using SimReal+ in classroom.

When asked whether SimReal+ can give students new possibilities in teaching mathematics in a flipped classroom setting, fourteen students expressed a real interest in this issue as this comment clearly reveals: "*Flipped classroom will give more prepared students and better discussions in the classroom*". Nevertheless, many students focused on out-of-class activities and use of videos rather than flipped classroom as a new method of teaching.

Pedagogical affordances at the mathematics subject level

In terms of pedagogical affordances at the mathematics subject level the most important issues addressed were the quality of mathematical content provided by SimReal+ and use of the tool to learn mathematics in teacher education. Table 9 shows that the most frequent item is the quality of mathematical content in terms of number of comments (39). This was highly valued by the students. The use of SimReal+ in secondary schools was also considered as important. These issues are supported by students' responses in their own words.

Table 9. Pedagogical affordances at the mathematics subject level

ITEM	NUMBER OF COMMENTS	NUMBER OF DISTINCT STUDENTS
QUALITY OF MATHEMATICS CONTENT		
• Design of mathematical tasks	12	12
• Congruence with paper-pencil mathematics	2	2
• Update and renew mathematics	2	2
• SimReal+ vs other digital tools	4	4
• Fidelity with mathematical concepts	10	10
• Supplement to lectures	9	9
USE OF SIMREAL+ TO LEARN MATHEMATICS IN TEACHER EDUCATION AT THE		
• Primary school level	6	6
• Middle school level	7	7
• Secondary school level	11	11

Firstly, most students agreed that SimReal+ has a high degree of mathematical content in terms of mathematical fidelity and correctness, and representation of mathematical properties and operations, e.g., formulas, functions, graphs, and geometrical figures. The following comment highlights this issue: "*SimReal+ presents the mathematics in a thorough and principally correct manner*", but as another student formulated, *"it can be done more, for example to solve an equation of second grad, (…), or to do more logical games, (…) exercises from number theory"*. Most students also believed that SimReal+ visualizations are easy to understand.

Secondly, many students think that the mathematical tasks are well-designed in terms of quality and that the advanced exercises were not difficult to understand. Nevertheless, the students were aware of the constraints of the tool as this rather critical comment shows: "*many tasks lack explanation or are not well-formulated, but once one understands what should be done, the exercises show a high degree of quality that promotes knowledge acquisition*".

Then, few students think that SimReal+ is better than GeoGebra, which has more advantages, such as an intuitive user interface and more differentiation. However, SimReal+ is considered as a good supplement to lectures. Other issues received few comments, e.g., congruence with paper-pencil mathematics.

Regarding the use of SimReal+ in teacher education, most students think that SimReal+ could be an appropriate tool in secondary schools and, in lesser degree, in middle schools, but not in primary schools. Finally, most students believed that SimReal+ enables teachers to concretize the mathematics subject curriculum.

WHAT ARE THE LIMITATIONS AND GAINS OF SIMREAL+ IN TEACHER EDUCATION?

The students were asked what they like and dislike about SimReal+ if the tool is given more importance in teacher education. Basically, most students reported both gains and limitations of SimReal+.

The gains of SimReal+ include variation of mathematical tasks, a wide range of mathematical topics ranging from basic to advanced mathematics, and the combination of simulations, animations, lesson streaming, and video lessons. The limitations of the tool are a lack of a user-friendly interface and navigation, insufficient feedback, and less motivation to use the tool in schools.

The students were asked to provide suggestions to make SimReal+ more appropriate for use in teacher education. They provided comments both on technological and pedagogical affordances (Table 10). The most frequent recommendation in terms of technological affordances is a better user interface for different type of users. In terms of pedagogical affordances, the students indicated a need for feedback, the possibility to design their own videos and visualizations, and more differentiation and individualization.

Table 10. Recommendations for improvements

ITEM	NUMBER OF COMMENTS	NUMBER OF DISTINCT STUDENTS
TECHNOLOGICAL AFFORDANCES	17	17
PEDAGOGICAL AFFORDANCES	22	22

DISCUSSION

The theoretical framework used in this work is an attempt to address affordances and constraints of SimReal+ in teacher education. The framework is useful to capture and assess the opportunities of SimReal+ in terms of technological and pedagogical affordances at different levels (Hammond, 2010). Furthermore, it was possible to adjust the notion of affordance to SimReal+ and address some other issues such as the adaptability of the tool to a teacher education context and the possibility of using the tool in a flipped classroom environment. The theoretical framework also provides the necessary knowledge to assist designers and teachers in improving existing functionalities and introducing new features according to the affordance perspective, including mathematical affordances (Watson, 2004, 2007).

Nevertheless, the research literature reveals that there is a lack of agreement on the elements of affordances to be considered in mathematics education, and, as a result, it is difficult to carry out empirical research to evaluate affordances of digital tools (Burlamaqui, & Dong, 2015). In addition, digital tools provide not only affordances, but also constraints that need to be identified. Furthermore, cultural affordances may also play an important role in teacher education, and these are particularly important in mathematics education (Chiappini, 2012). Furthermore, the notion of tool in mathematics education needs to be reconceptualised to better explore affordances and constraints of digital tools (Monaghan, Trouche, & Borwein, 2016). A reconceptualization needs to take in consideration new theories such as Actor-Network Theory (ANT), which does not consider technology simply as a tool, but rather as an actor that serves to reorganize human thinking (Latour, 2005).

Although this work does not aim to capture the full potential of the concept of affordances, it is possible to make reasonable interpretations of the results and draw some recommendations for using SimReal+ in teacher education. Firstly, in terms of technological affordances, the user interface of SimReal+ must be simplified to make it more intuitive and easy to use in teacher education. The notion of technological affordances is a self-evident requirement for any digital tool in mathematics education. In many cases, however, the impact of technological affordances on learning is limited when it comes to pedagogical use of the tool in authentic educational contexts. In addition, the functionality of a particular tool does not always result in pedagogical opportunities, which will only be visible when an explicit pedagogy guides the design and use of the tool in classroom (Burden, & Atkinson, 2008). Therefore, the concept of affordances has been expanded to include pedagogical affordances, such as learner autonomy, collaboration, variation, motivation, differentiation, and feedback. It is evident that both aspects of affordances are closely related to each other, and even congruent (Hadjerrouit, 2010; Nokelainen, 2006). Clearly, technological and pedagogical affordances are the very basis of any digital tool in teacher education, where didactical issues are of crucial importance.

Secondly, the results indicate that SimReal+ is technologically well designed in terms of accessibility and management facilities, but the tool is still not sufficiently user-friendly. Another positive side of SimReal+ is that it covers a wide range of mathematical content with varied levels of difficulty, including advanced, elementary, and basic mathematics. Likewise, the content is mathematically correct and reflects the underlying properties of ideal mathematics at different levels. These are opportunities that can be exploited by teachers to adequately adapt SimReal+ to a teacher education context.

Furthermore, SimReal+ seems to be pedagogically usable at the student level in terms of motivation, supplement to lectures and textbooks, variation in teaching, and multiple representations of mathematics. But still, work needs to be done to adjust the tool to individual students with different knowledge background. Moreover, SimReal+ does not provide assessment capabilities that may increase the learning process. Consequently, many students are still not ready to use the tool at their own will for learning mathematics.

Finally, the use of SimReal+ in teacher education indicates that the tool shows potential for teaching and learning mathematics that is suited to the students' knowledge level, although not all criteria are equally met. SimReal+ in its present form can be used to learn mathematics, but not on a regular basis, because it is still not fully appropriate for use in teacher education, unless the technological affordances are improved to make the user interface more intuitive and easy to use. Likewise, there is a need for more didactical functionalities and pedagogical modalities in terms of assessment, differentiation and individualisation in order for SimReal+ to become an integral part in teacher education.

Summarizing, SimReal+ has mathematical potential in teacher education, but the user interface is still not intuitive and user-friendly. Teachers also need to take into account the pedagogical dimension of teaching and learning mathematics in order to adapt SimReal+ to the context of teacher education.

CONCLUSION

The purpose of this work is to assess the affordances and constraints of SimReal+ in teacher education by asking students to respond to a survey questionnaire and open-ended questions. In addition, the students had the opportunity to comment the statements of the survey in their own words. This provided an important amount of information that gave a better sense and interpretation of the outcomes of the quantitative data. Even though, the results achieved are in line with those from the first experiment with SimReal+ in 2014 (Hadjerrouit, 2015), it is not possible to generalize the findings because of the small sample size (*N=22*). It is nevertheless possible to make sense of the results. In future research work, students' recommendations will be considered to improve the teaching of mathematics with SimReal+. In terms of technological affordances, there is a need for a better and intuitive user interface and navigation for different types of users. In terms of pedagogical affordances, there is a need for feedback and review modes, more differentiation and individualization, including the possibility of programming their own videos and visualizations. Moreover, the theoretical framework has proven to be useful to address the affordances of SimReal+ in teacher education. The framework will be refined and improved to better suit the next cycle of experimentation with SimReal+. Future research will also address issues that did not receive sufficient attention in this work such as metacognition, conceptual understanding of mathematics, and cultural affordances.

Finally, the survey questionnaire will be improved and supplementary data collection methods will be used to ensure more validity and reliability.

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Biography

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